Difficulty Controllable and Scalable Constrained Multi-objective Test Problems

Zhun Fan
Department of Electronic Engineering
Shantou University
Guangdong, Shantou 515063

Wenji Li
Department of Electronic Engineering
Shantou University
Guangdong, Shantou 515063

Xinye Cai
College of Computer Science and Technology, Nanjing University of Aeronautics and Astronautics
Jiangsu, Nanjing 210016

Huibiao Lin
Department of Electronic Engineering
Shantou University
Guangdong, Shantou 515063

Kaiwen Hu
Department of Electronic Engineering
Shantou University
Guangdong, Shantou 515063

Haibin Yin
School of Mechanical and Electronic Engineering, Wuhan University of Technology
Wuhan, Hubei 430070

***Abstract*—In this paper, we propose a general toolkit to construct constrained multi-objective optimisation problems (CMOPs) with three different kinds of constraint functions. Based on this toolkit, we suggest eight constrained multi-objective optimisation problems named CMOP1-CMOP8. As the ratio of feasible regions in the whole search space determines the difficulty of a constrained multi-objective optimisation problem, we propose four test instances CMOP3-6, which have very low ratio of feasible regions. To study the difficulties of proposed test instances, we make some experiments with two popular CMOEAs - MOEA/D-CDP and NSGA-II-CDP, and analysed their performances.**

***Index Terms*—Constrained Multi-objective Evolutionary Algorithm, Constrained Multi-objective Optimisation problem.**

I. Introduction

In engineering optimisation problems, most of them usually involve simultaneous optimisation of multiple and conflict objectives with severe constraints. Without loss of generality, an constrained multi-objective optimisation problem (CMOP) can be defined as follows:

$$\begin{aligned} \text{minimize} \quad & \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), \ldots, f_m(\mathbf{x}))^T \\ \text{subject to} \quad & g_i(\mathbf{x}) \geq 0, i = 1, \ldots, q \\ & h_j(\mathbf{x}) = 0, j = 1, \ldots, p \\ & \mathbf{x} \in \Omega \end{aligned} \tag{1}$$

where $\mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x}))^T \in \mathbb{R}^m$ is a m-dimensional objective vector, $g_i(\mathbf{x}) \geq 0$ define q inequality constraints, $h_j(\mathbf{x}) = 0$ define p equality constraints. $\Omega = \prod_{i=1}^{n}[a_i, b_i] \subseteq \mathbb{R}^n$ is the decision space, and $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$ is a n-dimensional decision variable. A solution $\mathbf{x}$ is said to be feasible if it meets $g_i(\mathbf{x}) \geq 0, i = 1, \ldots, q$ and $h_j(\mathbf{x}) = 0, j = 1, \ldots, p$ at the same time. For two feasible solution $\mathbf{x}^1$ and $\mathbf{x}^2$, solution $\mathbf{x}^1$ is said to dominate $\mathbf{x}^2$ if $f_i(\mathbf{x}^1) \leq f_i(\mathbf{x}^2)$ for each $i \in \{1, \ldots, m\}$ and $f_j(\mathbf{x}^1) \leq f_j(\mathbf{x}^2)$ for at least $j \in \{1, \ldots, m\}$, denotes as $\mathbf{x}^1 \preceq \mathbf{x}^2$. For a feasible solution $\mathbf{x}^* \in \Omega$, if there is no other feasible solution $\mathbf{x} \in \Omega$ dominating $\mathbf{x}^*$, $\mathbf{x}^*$ is said to be feasible Pareto optimal solution. The set of all the feasible Pareto optimal solutions is called the Pareto optimal set(PS). Mapping the PS into the objective space obtains a set of objective vectors, denotes as Pareto front (PF), where $PF = \{\mathbf{F}(\mathbf{x}) \in \mathbb{R}^m | \mathbf{x} \in PS\}$.

At present, most researchers concentrate on unconstrained multi-objective evolutionary algorithms (UMOEAs) and have achieved rapid research progress in them. Usually, UMOEAs can be divided into three categories: Pareto-dominance (e.g., NSGA-II[1], PAES-II[2] and SPEA-II[3]), decomposition based (e.g., MOEA/D[4], MOEA/D-DE[5], MOEA/D-M2M[6] and EAG-MOEA/D[7]), and indicator based methods (e.g., IBEA[8], R2-IBEA[9],SMS-EMOA[10] and HypE[11]). However, not enough attentions have been paid on constrained multi-objective evolutionary algorithms (CMOEAs). Actually, constraints greatly increase the difficulty of multi-objective optimisation problems, especially the ones with nonlinear equality or unequality constraints. As far as we know, most existing benchmark problems for multiobjective optimisation are unconstrained, such as ZDT[12] , DTLZ[13] and WFG[14]. Only two test suites (CTP[15] and CF[16]) are designed for constrained multi-objective optimization problems. In addition, the feasible ratio for both test suites are high and uncontrollable. The construction of new test suites for CMOPs thus becomes very necessary.

The rest of this paper is organised as follows. Section II introduces the construction method for generating CMOPs. Section III suggests a set of test problems. In the following Section VI, we compare the performance of two CMOEAs i.e., MOEA/D-CDP and NSGA-II-CDP by experimental study, and Section V concludes the paper.

II. Construction Toolkit

As we all known, constrained multi-objective optimisation problems include two parts, one part is the objective function and the other is constraint function. In order to facilitate analysing the Pareto front and Pareto set of constrained multi-objective optimisation problems, it is necessary to make some necessary assumptions. In terms of objective functions, Li, et al [17] suggested a general framework for constructing objective function of multi-objective optimisation problems as follows:

$$f_i(\mathbf{x}) = \alpha_i(x_{1:m-1}) + \beta_i(x_{1:m-1}, x_{m:n}) \tag{2}$$

where $x_{1:m-1} = (x_1, \ldots, x_m - 1)^T, x_{m:n} = (x_m, \ldots, x_n)^T$ are two sub-vectors of $x = (x_1, \ldots, x_n)^T$. The function $\alpha_i(x_{1:m-1})$ is called shape function, and $\beta_i(x_{1:m-1}, x_{m:n})$ is called nonnegative distance function. The objective function $f_i(x), i = 1, \ldots, m$ is the sum of shape function $\alpha_i(x_{1:m-1})$ and nonnegative distance function $\beta_i(x_{1:m-1}, x_{m:n})$. We adopt this general framework as the objective function of CMOPs. It is worth noting that this general framework is easy to scale to different Pareto front. Almost all of the existing unconstrained multi-objective optimisation problems have objective functions that can be expressed by Formula 2.

In terms of constraint functions, we define three basic types of constraint functions. As the equality constraint can be transformed into inequality constraint, here we only consider the unequality type. The first type of constraint function is defined as follows:

$$\Phi(x_{1:m-1}) \geq 0 \tag{3}$$

In order to facilitate description, we denote Formula 3 as Type-I. The constraint of Type-I only limits the sub-vector $x_{1:m-1}$ which decides the shape of Pareto front. In another word, the constraint of Type-I can change the shape of Pareto front. More specifically, the Pareto front with Type-I constraint is a subset of the Pareto front without constraints. It it worth noting that the ratio of feasible area in the search space can be controlled by setting the sub-vector $x_{1:m-1}$ into a small range. For example, we can define a constrained multi-objective optimisation problem as follows:

$$\begin{cases} \text{minimize} & f_1(x) = x_1 + g(x) \\ \text{minimize} & f_2(x) = 1 - x_1^2 + g(x) \\ & g(x) = \sum_{i=2}^{n} \left(x_i - sin(0.5\pi x_1)\right)^2 \\ \text{subject to} & c(x) = sin(a\pi x_1) - b \geq 0 \\ & x_i \in [0, 1] \end{cases} \tag{4}$$

where $a > 0$, $b \in [0, 1]$, in order to facilitate the drawing of the feasible area, we set $a = 10$, $b = 0.5$ and $n = 2$. The Pareto front without constraint and the Pareto front with Type-I constraint are shown in the left of Figure 1. The ratio of feasible area can be controlled by parameters a and b, if $b = 1.0$, x_1 will be limited to some discrete values, as shown in the right of Figure 1, and the number of discrete values of x_1 is controlled by a.

The second type of constraint function is defined as follows:

$$\Psi(x_{1:m-1}, x_{m:n}) \geq 0 \tag{5}$$

we represent this constraint function as Type-II. This constraint function limits the nonnegative distance function $\beta_i(x_{1:m-1}, x_{m:n})$ and decides the feasible ratio in the whole search space. So, we can obtain some CMOPs with low ratio of feasible solutions by restricting $\beta_i(x_{1:m-1}, x_{m:n})$ in a narrowed range . For example, we can define a CMOP as follows:

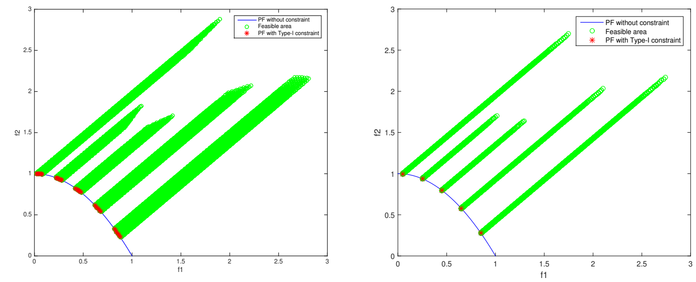

Fig. 1. Illustrations on the influence of Type-I constraint.

$$\begin{cases} \text{minimize} & f_1(x) = x_1 + g(x) \\ \text{minimize} & f_2(x) = 1 - x_1^2 + g(x) \\ & g(x) = \sum_{i=2}^{n} \left(x_i - sin(0.5\pi x_1)\right)^2 \\ \text{subject to} & c_1(x) = g(x) - a \geq 0 \\ & c_2(x) = b - g(x) \geq 0 \\ & n = 30, x_i \in [0, 1] \end{cases} \tag{6}$$

where $a \geq 0$,$b \geq 0$ and $b \geq a$. The Pareto front without constraint, the Pareto front with Type II constraint and the feasible area in the objective space are shown in Figure 2. It is worth noting the feasible area can be controlled by parameters a and b. If $b - a = 0.01$, the feasible area is very small as shown in the left of Figure 2, if $b - a = 0.1$, the feasible area is bigger than that of $b - a = 0.01$ as shown in the right of Figure 2.

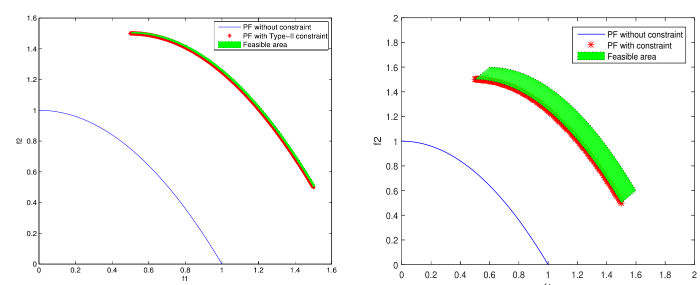

Fig. 2. Illustrations on the influence of Type-II constraint.

The third type of constraint function is defined as follows:

$$\Theta(f_{1:m}) \geq 0 \tag{7}$$

where $f_{1:m} = (f_1, \ldots, f_m)^T$, we denote this constraint function as Type-III. It simultaneously influences the shape of Pareto front and the ratio of feasible solutions in the whole search space. For example, in [18], we design a set of constraints in the objective space, the infeasible area is shown in the left of Figure 3. It is worth noting that we can obtain different kinds of constraint shapes in the objective space by setting parameters as shown in the right of Figure 3.

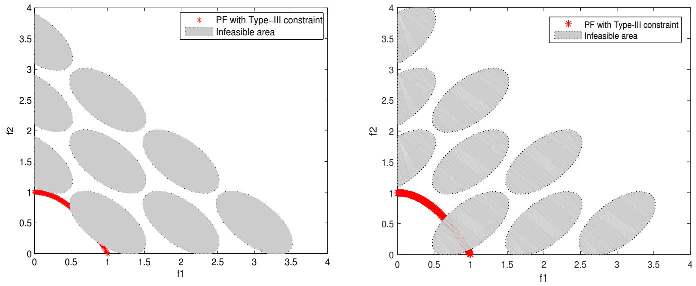

Fig. 3. Illustrations on the influence of Type-III constraint.

From the above definition of constraint functions, it is easy to tell that all of CTP[15] test instances belong to Type-III. For CF [16] test instances, CF1-CF3 and CF8-CF10 belong to Type-III, and CF4-CF7 belong to Type-II. It is worth noting that the above three types of constraint functions can be easy to scale to difficulty controllable CMOPs.

III. SCALABLE AND CONTROLLABLE CONSTRAINED MULTI-OBJECTIVE OPTIMISATION PROBLEMS

In this section, we proposed eight test instances with different kinds of constraint functions - CMOP1-CMOP8 using the above toolkit.

- Test Problem 1 - CMOP1:

$$\begin{cases} \text{minimize} & f_1(x) = x_1 + g_1(x) \\ \text{minimize} & f_2(x) = 1 - x_1^2 + g_2(x) \\ & g_1(x) = \sum_{j \in J_1} (x_j - sin(0.5\pi x_1))^2 \\ & g_2(x) = \sum_{j \in J_2} (x_j - cos(0.5\pi x_1))^2 \\ \text{subject to} & c(x) = sin(a\pi x_1) - 0.5 \geq 0 \\ & J_1 = \{j|j \text{ is odd and } 2 \leq j \leq n\} \\ & J_2 = \{j|j \text{ is even and } 2 \leq j \leq n\} \\ & a = 20, n = 30, x_j \in [0, 1] \end{cases} \tag{8}$$

Here, the parameter a controls the number of discrete segments. A smaller a will generate fewer segments. The constraint function belongs to Type-I, and the Pareto front of CMOP1 is discrete and concave as shown in Figure 4.

- Test Problem 2 - CMOP2:

$$\begin{cases} \text{minimize} & f_1(x) = x_1 + g_1(x) \\ \text{minimize} & f_2(x) = 1 - \sqrt{x_1} + g_2(x) \\ & g_1(x) = \sum_{j \in J_1} (x_j - sin(0.5\pi x_1))^2 \\ & g_2(x) = \sum_{j \in J_2} (x_j - cos(0.5\pi x_1))^2 \\ \text{subject to} & c(x) = sin(a\pi x_1) - 0.5 \geq 0 \\ & J_1 = \{j|j \text{ is odd and } 2 \leq j \leq n\} \\ & J_2 = \{j|j \text{ is even and } 2 \leq j \leq n\} \\ & a = 20, n = 30, x_j \in [0, 1] \end{cases} \tag{9}$$

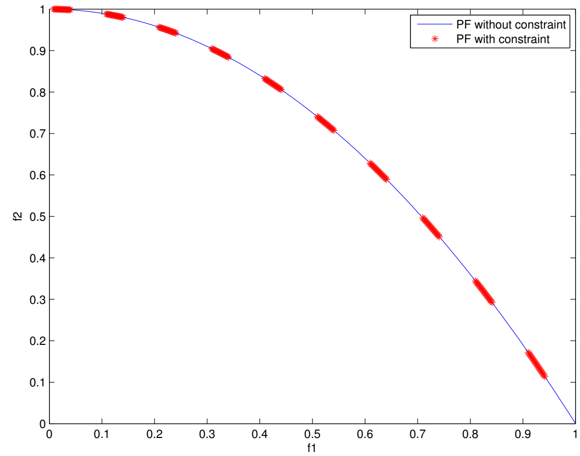

Fig. 4. The Pareto front of CMOP1

Test instance CMOP2 is similar to CMOP1 and the difference between them is the shape of Pareto front. The Pareto front of CMOP2 is discrete and convex as shown in Figure 5.

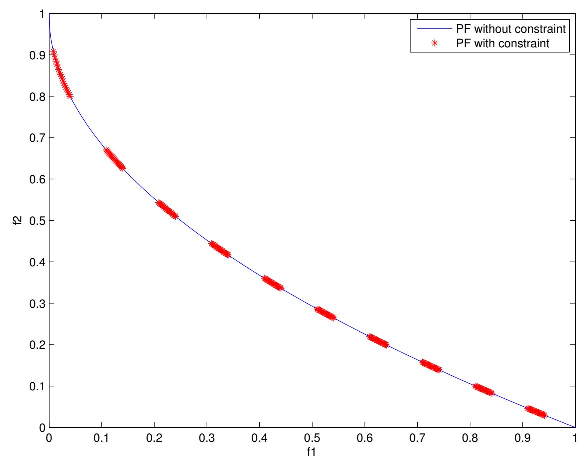

Fig. 5. The Pareto front of CMOP2

- Test Problem 3 - CMOP3:

$$\begin{cases} \text{minimize} & f_1(x) = x_1 + g_1(x) \\ \text{minimize} & f_2(x) = 1 - x_1^2 + g_2(x) \\ & g_1(x) = \sum_{j \in J_1} (x_j - sin(0.5\pi x_1))^2 \\ & g_2(x) = \sum_{j \in J_2} (x_j - cos(0.5\pi x_1))^2 \\ \text{subject to} & c_1(x) = (a - g_1(x)) * (g_1(x) - b) \geq 0 \\ & c_2(x) = (a - g_2(x)) * (g_2(x) - b) \geq 0 \\ & J_1 = \{j|j \text{ is odd and } 2 \leq j \leq n\} \\ & J_2 = \{j|j \text{ is even and } 2 \leq j \leq n\} \\ & a = 0.51, b = 0.5, n = 30, x_j \in [0, 1] \end{cases} \tag{10}$$

The constraint functions of CMOP3 belong to Type-II. Where, a and b control the ratio of feasible solutions in the whole search space, and $a >= b$. The smaller value of $a - b$ will generates a lower ratio of feasible solutions. The Pareto front of CMOP3 is concave as shown in Figure 6.

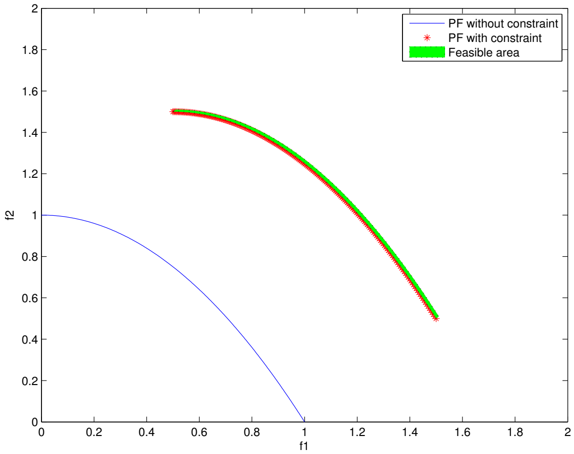

Fig. 6. The Pareto front of CMOP3

- Test Problem 4 - CMOP4:

$$\begin{cases} \text{minimize} & f_1(x) = x_1 + g_1(x) \\ \text{minimize} & f_2(x) = 1 - \sqrt{x_1} + g_2(x) \\ & g_1(x) = \sum\limits_{j \in J_1} (x_j - sin(0.5\pi x_1))^2 \\ & g_2(x) = \sum\limits_{j \in J_2} (x_j - cos(0.5\pi x_1))^2 \\ \text{subject to} & c_1(x) = (a - g_1(x)) * (g_1(x) - b) \geq 0 \\ & c_2(x) = (a - g_2(x)) * (g_2(x) - b) \geq 0 \\ & J_1 = \{j | j \text{ is odd and } 2 \leq j \leq n\} \\ & J_2 = \{j | j \text{ is even and } 2 \leq j \leq n\} \\ & a = 0.51, b = 0.5, n = 30, x_j \in [0, 1] \end{cases} \tag{11}$$

CMOP4 test instance is transformed from CMOP3, and the Pareto front of CMOP4 is convex as shown in Figure 7.

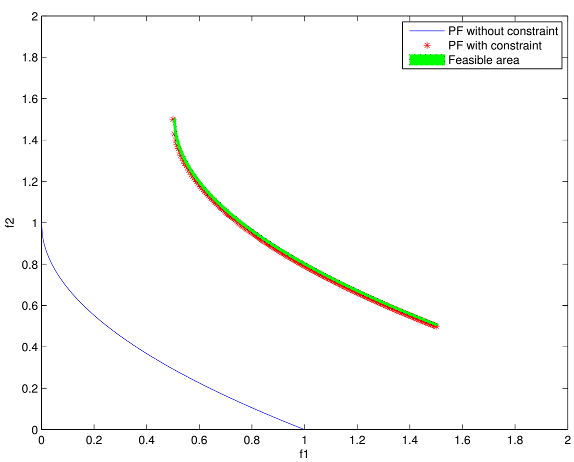

Fig. 7. The Pareto front of CMOP4

- Test Problem 5 - CMOP5:

$$\begin{cases} \text{minimize} & f_1(x) = x_1 + g_1(x) \\ \text{minimize} & f_2(x) = 1 - x_1^2 + g_2(x) \\ & g_1(x) = \sum\limits_{j \in J_1} (x_j - sin(0.5\pi x_1))^2 \\ & g_2(x) = \sum\limits_{j \in J_2} (x_j - cos(0.5\pi x_1))^2 \\ \text{subject to} & c_1(x) = (a - g_1(x)) * (g_1(x) - b) \geq 0 \\ & c_2(x) = (a - g_2(x)) * (g_2(x) - b) \geq 0 \\ & c_3(x) = sin(c\pi x_1) - 0.5 \geq 0 \\ & J_1 = \{j | j \text{ is odd and } 2 \leq j \leq n\} \\ & J_2 = \{j | j \text{ is even and } 2 \leq j \leq n\} \\ & a = 0.51, b = 0.5, c = 20, n = 30, x_j \in [0, 1] \end{cases} \tag{12}$$

The constraint functions of CMOP5 are hybrid with Type-I and Type-II, and the feasible area in objective space is very small. We can set the three parameters a, b and c to generate difficulty controllable test instances. The Pareto front of CMOP5 is discrete and concave as shown in Figure 8.

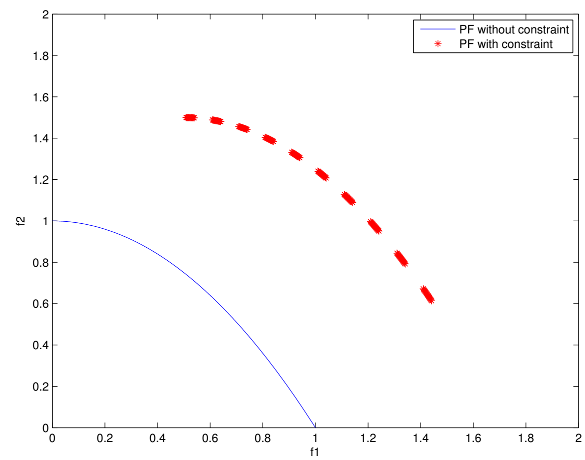

Fig. 8. The Pareto front of CMOP5

- Test Problem 6 - CMOP6:

$$\begin{cases} \text{minimize} & f_1(x) = x_1 + g_1(x) \\ \text{minimize} & f_2(x) = 1 - \sqrt{x_1} + g_2(x) \\ & g_1(x) = \sum\limits_{j \in J_1} (x_j - sin(0.5\pi x_1))^2 \\ & g_2(x) = \sum\limits_{j \in J_2} (x_j - cos(0.5\pi x_1))^2 \\ \text{subject to} & c_1(x) = (a - g_1(x)) * (g_1(x) - b) \geq 0 \\ & c_2(x) = (a - g_2(x)) * (g_2(x) - b) \geq 0 \\ & c_3(x) = sin(c\pi x_1) - 0.5 \geq 0 \\ & J_1 = \{j | j \text{ is odd and } 2 \leq j \leq n\} \\ & J_2 = \{j | j \text{ is even and } 2 \leq j \leq n\} \\ & a = 0.51, b = 0.5, c = 20, n = 30, x_j \in [0, 1] \end{cases} \tag{13}$$

CMOP6 test instance is transformed from CMOP5, and the Pareto front of CMOP6 is discrete and convex as shown in Figure 9.

- Test Problem 7 - CMOP7:

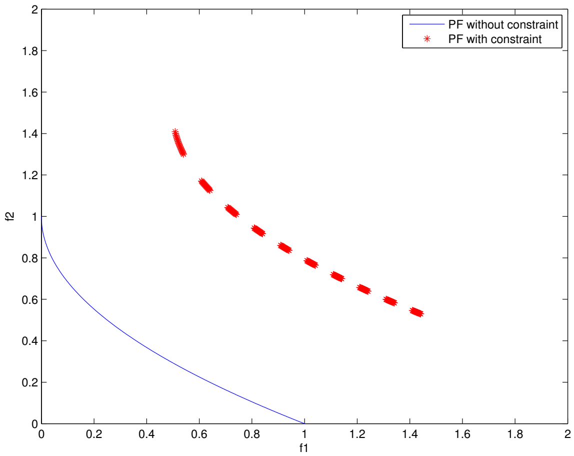

Fig. 9. The Pareto front of CMOP6

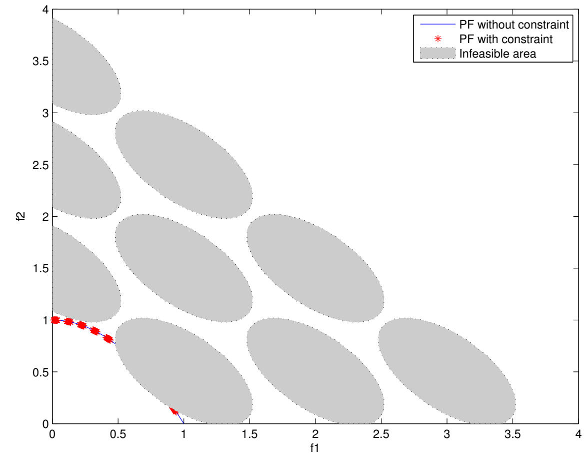

Fig. 10. The Pareto front of CMOP7

$$\begin{cases} \text{minimize} & f_1(x) = x_1 + g_1(x) \\ \text{minimize} & f_2(x) = 1 - x_1^2 + g_2(x) \\ & g_1(x) = \sum_{j \in J_1} (x_j - sin(0.5\pi x_1))^2 \\ & g_2(x) = \sum_{j \in J_2} (x_j - cos(0.5\pi x_1))^2 \\ \text{subject to} & c_k(x) = ((f_1 - p_k)cos\theta - (f_2 - q_k)sin\theta)^2/a^2 \\ & +((f_1 - p_k)sin\theta + (f_2 - q_k)cos\theta)^2/b^2 \geq 1 \\ & c_{10}(x) = sin(c\pi x_1) - 0.5 \geq 0 \\ & p = [0, 1, 0, 1, 2, 0, 1, 2, 3] \\ & q = [1.5, 0.5, 2.5, 1.5, 0.5, 3.5, 2.5, 1.5, 0.5] \\ & J_1 = \{j | j \text{ is odd and } 2 \leq j \leq n\} \\ & J_2 = \{j | j \text{ is even and } 2 \leq j \leq n\} \\ & a^2 = 0.1, b^2 = 0.4, \theta = -0.25\pi \\ & c = 20, n = 30, x_j \in [0, 1], k = 1, \ldots, 9 \end{cases} \tag{14}$$

The constraint functions of test instance CMOP7 are hybrid with Type-I and Type-III. The parameter c controls the number of discrete segments. Constraint functions $c_k(x)$ construct k ellipsoids in the objective space and each $c_k(x)$ consists of five parameters $(\theta_k, a_k, b_k, p_k, q_k)$. Here, all of the ellipsoids have the same long axis a, minor axis b and counterclockwise rotation angle θ. We can adjust the centre of each ellipsoid by setting p and q. It is worth noting that we can obtain many different kinds of constraint shapes in the objective space by tuning five parameters $(\theta_k, a_k, b_k, p_k, q_k)$ as shown in Figure 3 . The Pareto front of CMOP7 is discrete and concave as shown in Figure 10.

- Test Problem 8 - CMOP8:

$$\begin{cases} \text{minimize} & f_1(x) = x_1 + g_1(x) \\ \text{minimize} & f_2(x) = 1 - \sqrt{x_1} + g_2(x) \\ & g_1(x) = \sum_{j \in J_1} (x_j - sin(0.5\pi x_1))^2 \\ & g_2(x) = \sum_{j \in J_2} (x_j - cos(0.5\pi x_1))^2 \\ \text{subject to} & c_k(x) = ((f_1 - p_k)cos\theta - (f_2 - q_k)sin\theta)^2/a^2 \\ & +((f_1 - p_k)sin\theta + (f_2 - q_k)cos\theta)^2/b^2 \geq 1 \\ & c_{10}(x) = sin(c\pi x_1) - 0.5 \geq 0 \\ & p = [0, 1, 0, 1, 2, 0, 1, 2, 3] \\ & q = [1.5, 0.5, 2.5, 1.5, 0.5, 3.5, 2.5, 1.5, 0.5] \\ & J_1 = \{j | j \text{ is odd and } 2 \leq j \leq n\} \\ & J_2 = \{j | j \text{ is even and } 2 \leq j \leq n\} \\ & a^2 = 0.1, b^2 = 0.4, \theta = -0.25\pi \\ & c = 20, n = 30, x_j \in [0, 1], k = 1, \ldots, 9 \end{cases} \tag{15}$$

CMOP8 test instance is transformed from CMOP7 and the Pareto front of CMOP8 is discrete and convex as shown in Figure 11. It is worth noting that for CMOP7-8, we only draw the infeasible area of Type-III constraint in the objective space. Because the Type-I constraint is very difficult to depict in the objective space, especially, when the dimension of decision variables is very large.

IV. EXPERIMENTAL STUDY

A. Experimental Settings

To verify the difficulties of the suggested test instances in the Section III, we applied two commonly used CMOEAs (i.e., MOEA/D-CDP and NSGA-II-CDP) in our experiments. The detailed parameter settings are summarised as follows.

1) Setting for reproduction operators: The mutation probability $Pm = 1/n$ (n is the number of decision variables) and its distribution index is set to be 20. For the DE operator, we set $CR = 1.0$ and $f = 0.5$.

2) Population size: $N = 200$.

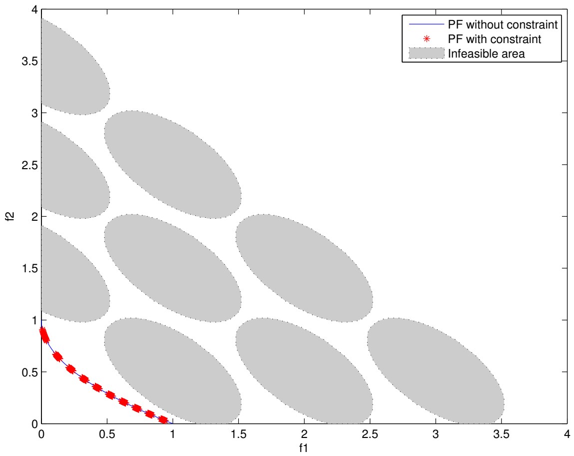

Fig. 11. The Pareto front of CMOP8

3) Number of runs and stopping condition: Each algorithm runs 30 times independently on each test problems. The algorithm stops until 300 000 function evaluations.

4) Neighborhood size: $T = 20$.

5) Probability use to select in the neighbourhood: $\delta = 0.9$.

6) The maximal number of solutions replaced by a child: $nr = 2$.

B. Performance Metric

To measure the performance of MOEA/D-CDP and NSGA-II, we use two metrics - inverted generation distance (IGD)[19] and relative hypervolume indicator (I_H^-) [20] and the detail definitions of IGD and I_H^- are given as follows:

- **Inverted Generational Distance** (IGD):

The IGD metric simultaneously reflects the performance of convergence and diversity, and it is defined as follows:

$$\begin{cases} IGD(P^*, A) = \frac{\sum\limits_{y^* \in P^*} d(y^*, A)}{\|P^*\|} \\ d(y^*, A) = \min\limits_{y \in A} \{ \sqrt{\sum_{i=1}^{m} (y_i^* - y_i)^2} \} \end{cases} \tag{16}$$

Where P^* is the ideal Pareto front set, A is an approximate Pareto front set achieved by evolutionary multi-objective algorithm. IGD metric denotes the distance between P^* and A.

- **Relative Hypervolume Indicator** (I_H^-):

I_H^- simultaneously considers the distribution of the obtained Pareto front A and its vicinity to the true Pareto front. $I_H(P^*, R)$ is defined as the volume enclosed by P^* and the reference vector $R = (R_1, \ldots, R_m)$. $I_H(A, R)$ is defined as the volume enclosed by A and the reference vector R. $I_H^-(A, P^*, R)$can be defined as:

$$\begin{cases} I_H^-(A, P^*, R) = I_H(P^*, R) - I_H(A, R) \\ I_H(P^*, R) = Vol_{v \in P^*}(v) \\ I_H(A, R) = Vol_{v \in A}(v) \end{cases} \tag{17}$$

Here, $Vol_{v \in P^*}(v)$ represents the volume enclosed by solution $v \in P^*$ and the reference vector R, and $Vol_{v \in A}(v)$ represents the volume enclosed by solution $v \in A$ and the reference vector R. When computing the above metrics, 200 points are uniformly sampled from the true PF. The reference point R is $(1.2, 1.2)^T$ for CMOP1, CMOP2, CMOP7 and CMOP8. For CMOP3-CMOP6, the reference point R is set to $(1.6, 1.6)^T$. It is worth noting that the smaller values of IGD and I_H^- represent the better performance of both diversity and convergency.

C. Experimental Results and Discussions

In order to demonstrate the difficulty levels of the suggested CMOPs, we test them by using two classic CMOEAs - MOEA/D-CDP and NSGA-II-CDP. The final population with the best I_H^- metric in 30 independent runs are shown in Figure 12. From Figure 12, we can observe that MOEA/D-CDP has obtained better Pareto fronts on CMOP1, CMOP2, CMOP7 and CMOP8 than NSGA-II-CDP. For CMOP3, CMOP4, CMOP5 and CMOP6, MOEA/D-CDP and NSGA-II-CDP have similar Pareto fronts.

The mean values of IGD and I_H^- are shown in Table I and Table II, and Wilcoxon's rank sum test values of IGD and I_H^- are at a 0.05 significance level. The IGD metrics of MOEA/D-CDP on CMOP1, CMOP2, CMOP5, CMOP7 and CMOP8 are significant better than that of NSGA-II-CDP, and for CMOP4, NSGA-II-CDP is significant better than MOEA/D-CDP. In terms of I_H^- metric, MOEA/D-CDP is significant better than NSGA-II-CDP on CMOP1, CMOP2, CMOP7 and CMOP8, and significant worse than NSGA-II-CDP on test instance CMOP4.

For CMOP3-CMOP6, both methods only acquired a small part of the true Pareto front. The reason is that CMOP3-CMOP6 have very low ratio of feasible solutions. In the framework of MOEA/D-CDP, when an individual is feasible, it will quickly replace its neighbourhood and then extend to the whole population by using CDP mechanism. For NSGA-II-CDP, a feasible solution ranks in the first level, and this will also replace the infeasible solutions quickly by using CDP approach. As the feasible area is very narrow, it is very hard for the population to expand in the narrowed feasible area, and thus difficult to search for the optimum. From the above experimental analysis, we can conclude that the suggested test instances are not easy to solve by using the existing CMOEAs, and MOEA/D-CDP works better than NSGA-II-CDP on most test instances.

V. Conclusion

Constrained multi-objecitve optimisation problem with low ratio of feasible solutions is a difficult feature for many existing CMOEAs. In this work, we propose an construction toolkit to build difficulty controllable constrained multi-objective test problems. To verify the difficulties of suggested test instances, we make some experiments to test the performance of two popular CMOEAs - MOEA/D-CDP and NSGA-II-CDP. The experimental results show that the test instances with low ratio of feasible solutions in the whole search space is not easy to solve by these two algorithms. To enhance the performance of two classic CMOEAs - MOEA/D-CDP and NSGA-II-CDP

TABLE I
IGD VALUES OF MOEA/D-CDP AND NSGA-II-CDP.

Instance	MOEA/D-CDP		NSGA-II-CDP		Wilcoxon's Rank	
–	Mean	Std.	Mean	Std.	p-value	h-value
CMOP1	**6.46E-03**	6.68E-04	7.30E-01	2.94E-01	1.73E-06	**1.00E+00**
CMOP2	**5.78E-03**	9.52E-04	7.17E-01	3.23E-01	1.73E-06	**1.00E+00**
CMOP3	3.53E-01	9.84E-03	3.58E-01	8.71E-02	7.86E-02	0.00E+00
CMOP4	3.14E-01	8.88E-03	**2.90E-01**	3.82E-02	3.72E-05	**1.00E+00**
CMOP5	**3.49E-01**	1.84E-02	3.53E-01	7.37E-02	1.17E-02	**1.00E+00**
CMOP6	3.18E-01	6.06E-03	3.33E-01	6.87E-02	6.14E-01	0.00E+00
CMOP7	**1.99E-01**	1.90E-02	1.49E+00	2.06E-01	1.73E-06	**1.00E+00**
CMOP8	**4.25E-02**	6.36E-02	1.49E+00	3.07E-01	1.73E-06	**1.00E+00**

TABLE II
I_H^- VALUES OF MOEA/D-CDP AND NSGA-II-CDP.

Instance	MOEA/D-CDP		NSGA-II-CDP		Wilcoxon's Rank	
–	Mean	Std.	Mean	Std.	p-value	h-value
CMOP1	**6.13E-03**	7.67E-04	6.61E-01	8.16E-02	1.73E-06	**1.00E+00**
CMOP2	**7.35E-03**	2.64E-03	8.58E-01	1.92E-01	1.73E-06	**1.00E+00**
CMOP3	3.24E-01	1.41E-02	3.14E-01	3.92E-02	7.86E-02	0.00E+00
CMOP4	3.88E-01	2.43E-02	**3.66E-01**	5.93E-02	8.73E-03	**1.00E+00**
CMOP5	2.95E-01	1.44E-02	2.90E-01	2.90E-02	1.59E-01	0.00E+00
CMOP6	3.71E-01	1.97E-02	3.79E-01	6.47E-02	6.58E-01	0.00E+00
CMOP7	**2.79E-01**	7.89E-03	5.93E-01	0.00E+00	1.73E-06	**1.00E+00**
CMOP8	**8.42E-02**	1.31E-01	1.07E+00	2.75E-02	1.73E-06	**1.00E+00**

on suggested test instances in this work, more works need to be done in terms of improving CMOEAs' diversity and designing new constraint handling mechanisms. The future research work includes adopting external archives to enhance the diversity of existing CMOEAs.

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REFERENCES

[1] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *Evolutionary Computation, IEEE Transactions on*, vol. 6, no. 2, pp. 182–197, 2002.

[2] D. W. Corne, N. R. Jerram, J. D. Knowles, M. J. Oates *et al.*, "PESA-II: Region-based selection in evolutionary multiobjective optimization," in *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO2001)*. Citeseer, 2001.

[3] E. Zitzler, M. Laumanns, L. Thiele, E. Zitzler, E. Zitzler, L. Thiele, and L. Thiele, "SPEA2: Improving the strength pareto evolutionary algorithm," 2001.

[4] Q. Zhang and H. Li, "MOEA/D: A multiobjective evolutionary algorithm based on decomposition," *Evolutionary Computation, IEEE Transactions on*, vol. 11, no. 6, pp. 712–731, 2007.

[5] H. Li and Q. Zhang, "Multiobjective optimization problems with complicated pareto sets, MOEA/D and NSGA-II," *Evolutionary Computation, IEEE Transactions on*, vol. 13, no. 2, pp. 284–302, 2009.

[6] H.-L. Liu, F. Gu, and Q. Zhang, "Decomposition of a multiobjective optimization problem into a number of simple multiobjective subproblems," *Evolutionary Computation, IEEE Transactions on*, vol. 18, no. 3, pp. 450–455, 2014.

[7] X. Cai, Y. Li, Z. Fan, and Q. Zhang, "An external archive guided multiobjective evolutionary algorithm based on decomposition for combinatorial optimization," *Evolutionary Computation, IEEE Transactions on*, vol. 19, no. 4, pp. 508–523, Aug 2015.

[8] E. Zitzler and S. Künzli, "Indicator-based selection in multiobjective search," in *Parallel Problem Solving from Nature-PPSN VIII*. Springer, 2004, pp. 832–842.

[9] D. H. Phan and J. Suzuki, "R2-IBEA: R2 indicator based evolutionary algorithm for multiobjective optimization," in *Evolutionary Computation (CEC), 2013 IEEE Congress on*. IEEE, 2013, pp. 1836–1845.

[10] N. Beume, B. Naujoks, and M. Emmerich, "SMS-EMOA: Multiobjective selection based on dominated hypervolume," *European Journal of Operational Research*, vol. 181, no. 3, pp. 1653–1669, 2007.

[11] J. Bader and E. Zitzler, "HypE: An algorithm for fast hypervolume-based many-objective optimization," *Evolutionary computation*, vol. 19, no. 1, pp. 45–76, 2011.

[12] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler, *Scalable test problems for evolutionary multiobjective optimization*. Springer, 2005.

[13] K. Deb, L. Thiele, and M. Laumanns, "Scalable multi-

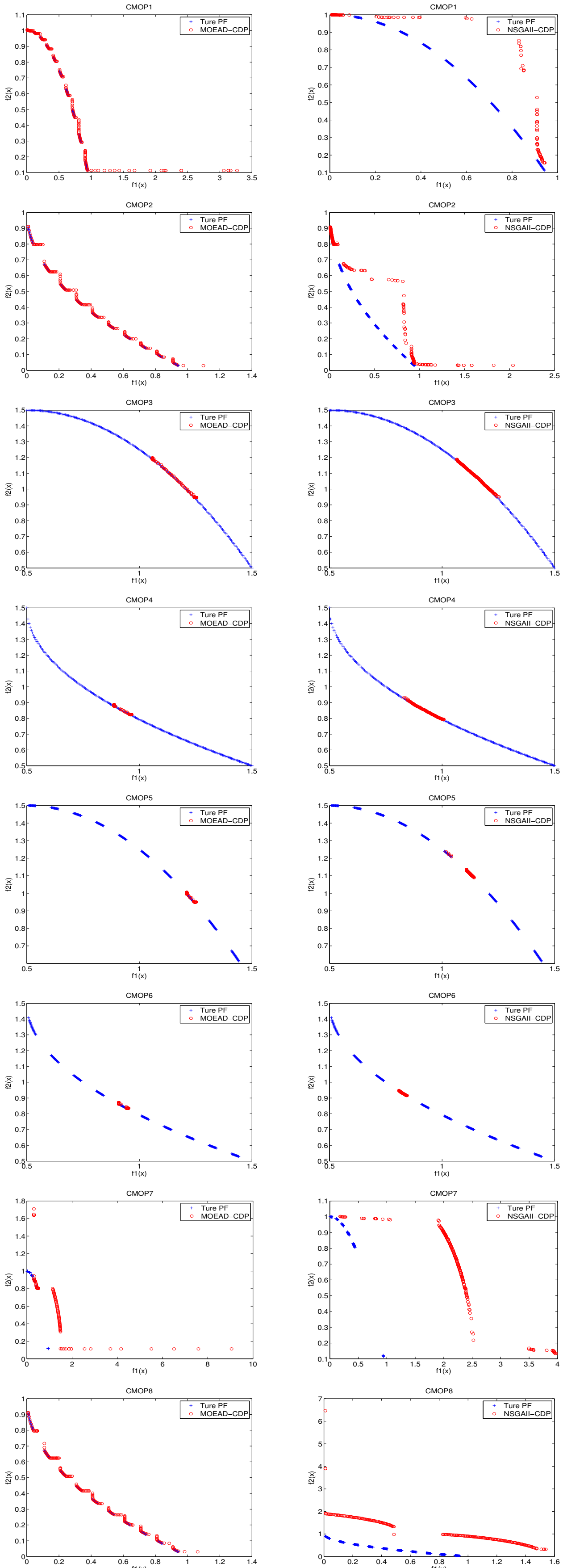

Fig. 12. The final populations with the best I_H^- metric in 30 independent runs using MOEA/D-CDP and NSGA-II-CDP

objective optimization test problems," in *Proceedings of the Congress on Evolutionary Computation (CEC-2002),(Honolulu, USA)*. Proceedings of the Congress on Evolutionary Computation (CEC-2002),(Honolulu, USA), 2002, pp. 825–830.

[14] S. Huband, P. Hingston, L. Barone, and L. While, "A review of multiobjective test problems and a scalable test problem toolkit," *Evolutionary Computation, IEEE Transactions on*, vol. 10, no. 5, pp. 477–506, 2006.

[15] K. Deb, *Multi-objective optimization using evolutionary algorithms*. John Wiley & Sons, 2001, vol. 16.

[16] Q. Zhang, A. Zhou, S. Zhao, P. N. Suganthan, W. Liu, and S. Tiwari, "Multiobjective optimization test instances for the cec 2009 special session and competition," *University of Essex, Colchester, UK and Nanyang technological University, Singapore, special session on performance assessment of multi-objective optimization algorithms, technical report*, pp. 1–30, 2008.

[17] H. Li, Q. Zhang, and J. Deng, "Multiobjective test problems with complicated pareto fronts: Difficulties in degeneracy," in *Evolutionary Computation (CEC), 2014 IEEE Congress on*. IEEE, 2014, pp. 2156–2163.

[18] Z. Fan, W. Li, X. Cai, H. Lin, S. Xie, and E. D. Goodman, "A new repair operator for multi-objective evolutionary algorithm in constrained optimization problems," *CoRR*, vol. abs/1504.00154, 2015. [Online]. Available: http://arxiv.org/abs/1504.00154

[19] P. A. Bosman and D. Thierens, "The balance between proximity and diversity in multiobjective evolutionary algorithms," *Evolutionary Computation, IEEE Transactions on*, vol. 7, no. 2, pp. 174–188, 2003.

[20] J. Knowles, L. Thiele, and E. Zitzler, "A tutorial on the performance assessment of stochastic multiobjective optimizers," *Tik report*, vol. 214, pp. 327–332, 2006.